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Virtual-tunneling-assisted vertical conduction in superlattices with intentional disorder

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Abstract. A new mechanism of vertical conduction in superlattices with intentional disorder is discussed. We show that at low temperatures the conductance of these structures can be mostly determined by phonon-assisted transitions between the second-nearest wells in the vicinity of the well with the highest size-quantization level. Such transitions involve virtual transitions to the state of the intermediate well and are characterized by a low activation energy and specific dependence of the vertical conductance on the scale of disorder.

1 Introduction

Superlattices with intentional disorder (SLID) and multiple quantum well structures, in which disorder was introduced by random controlled variations of the well widths in the process of structure deposition, were first discussed in [1] and were experimentally realized in [2, 3]. Vertical conduction (in the direction of the SLID growth axis) in such structures was studied by optical methods, in particular, stationary and picosecond luminescence spectroscopy, and also by direct measurements of the vertical conductance. The conductance measurements for Si-doped GaAs/GaAlAs SLID revealed some unusual features [4, 5]. Thus at low temperatures the temperature dependence of the conductance was practically nonactivated (quasimetallic) even for structures with large disorder, namely, when the width of the distribution of size quantization levels exceeded the estimated miniband width. Even though the Coulomb fields arising from the electron redistribution between the wells can give rise to an appreciable suppression of disorder (narrowing of the level distribution) [6], the explanation of the relatively weak temperature dependence of the conductance still remains problematic.

The wave functions of electronic states in SLID can be written in the form $\psi_{\lambda \mathbf{k}_\parallel} = NU_\lambda(z) \exp(i\mathbf{k}_\parallel \rho)$, where N is the normalization factor, z is the coordinate in the SLID growth direction, ρ is the in-plane position vector, $U_\lambda(z)$ is the eigenstate corresponding to the solution of the one-dimensional problem with the potential $V(z) = \sum_n V_n(z)$ describing the modulation of the conduction band edge, and $V_n(z)$ is the potential of the n th well (we set $V_n(z) = 0$ in the barrier regions). The functions $U_\lambda(z)$ are localized and for small overlap of wave functions of the neighboring wells, we can use the basis of "atomic-like" wave functions localized at the corresponding wells. Since for the structures of the type studied in [4] the contributions >from higher subbands are negligible, we can take only the lowest subband into account and write $\lambda = n$. For SLID the vertical conduction is usually controlled by phonon-assisted tunneling between neighboring wells. As in the standard hopping theory (e.g., see [7, 8]), the low-field problem can be reduced to the equivalent resistance network, with resistances expressed in terms of resulting transition rates between the wells (the difference from the standard hopping problem is that we have to sum over the initial and final states of the wells). Thus, for the resistance $R_{nn'}$ connecting the wells

n and n' we have

$$R_{nn'}^{-1} = (e^2/kT) \sum_{\mathbf{k}_\parallel, \mathbf{k}'_\parallel} W_{n\mathbf{k}_\parallel, n'\mathbf{k}'_\parallel} f_{n\mathbf{k}_\parallel}(1 - f_{n'\mathbf{k}'_\parallel}), \tag{1}$$

where $\mathbf{k}_\parallel$ is the two-dimensional momentum in the well plane, $W_{n\mathbf{k}_\parallel, n'\mathbf{k}'_\parallel}$ is the probability of phonon-assisted transitions from the state $n'\mathbf{k}'_\parallel$ to the state $n\mathbf{k}_\parallel$ and $f_{n\mathbf{k}_\parallel}$ is the equilibrium average occupation number of the state $n\mathbf{k}_\parallel$. If the higher level (say, E_n) lies above the Fermi level μ, then (for $E_n - \mu \gg kT$) we have $R_{nn'} = R_0 \exp\{-(E_n - \mu)/kT\}$, where the preexponential factor R_0 depends on the overlap of the wave functions of neighboring wells and only weakly depends on energy and temperature. In the nearest-neighbor approximation, the network is quasi-one-dimensional, and its resistance is given by the sum of series resistances connecting of the neighboring wells of the chain. Clearly, if the scatter of the energy levels is greater than the miniband width and kT, then the total resistance is determined by a critical well (or by a small number of critical wells) with adjoining largest resistances. These resistances exponentially depend on temperature and it follows that the temperature dependence of the total vertical conductance is activated, with the activation energy determined by the position of the highest energy levels relative to the Fermi level. However, with lowering temperature the vertical conductance can be controlled by parallel shunt resistances corresponding to second-neighbor phonon-assisted transitions. In fact, the critical resistances exponentially increase and the level separation for transitions between wells lying on different sides of the critical one (and hence the activation energy) is typically smaller for these transitions. We shall see that the most probable transitions involve virtual intermediate-well states.

2 Probability of second-neighbor phonon-assisted transitions via virtual states

Let us consider a three-well configuration representing the critical region consisting of the critical well 2 with a high level E_2 lying above the Fermi level and two adjacent wells 1 and 3 with lower levels. Next, let $u_n(z)$ and E_n be the wave functions and energies corresponding to the solution of the one-dimensional problem with a single n th well (we assume that $E_2 > E_3 > E_1$). Taking the overlap into account, we can construct the hybridized wave functions corresponding to the solution of the three-well problem. Thus, for level separations exceeding the transfer integrals $t_{mn} = \int dz\, u_m(z) \tilde{V}_n(z)\, u_n(z)$, where $\tilde{V}_n(z) = \sum_{n' \neq n} V_{n'}(z)$, the lowest energy state is mostly localized at well 1 and the corresponding wave function is

$$U_1(z) = N_1\{u_1(z) + c_{12}u_2(z)\}, \tag{2}$$

where N_1 is the normalization factor and $c_{12} = t_{12}/(E_2 - E_1)$. Here we have omitted the transfer integrals between wells 1 and 3 and as usually neglected the nonorthogonality integrals with the functions $u_n(z)$. Similarly, we can write out the expressions for the wave functions $U_2(z)$ and $U_3(z)$ localized at the wells 2 and 3.

The phonon-assisted transition probability, which involves the spatial displacement between wells 1 and 3 is expressed in terms of the matrix element

$$\begin{aligned} I_{3\mathbf{k}_\parallel, 1\mathbf{k}'_\parallel} &= \int dz U_1(z) H_{e,ph}^{\mathbf{k}_\parallel \mathbf{k}'} U_3(z) \\ &= N_1 N_3 \{c_{12} \int dz u_2(z) H_{e,ph}^{\mathbf{k}_\parallel \mathbf{k}'_\parallel} u_3(z) + c_{32} \int dz u_1(z) H_{e,ph}^{\mathbf{k}_\parallel \mathbf{k}'_\parallel} u_2(z)\}, \end{aligned} \tag{3}$$

where $H_{e,ph}^{\mathbf{k}_\parallel \mathbf{k}'_\parallel} = \int \mathbf{d}\rho \, \exp\{i(\mathbf{k}_\parallel - \mathbf{k}'_\parallel)\rho\} H_{e,ph}$ and $H_{e,ph}$ is the Hamiltonian of the electron-phonon interaction.

Expression (3) describes the amplitude of the transition between wells 1 and 3 as the sum of contributions of the two channels, the first one corresponding to the succession of virtual tunneling from 1 to 2 succeeded by phonon-assisted transition from 2 to 3 and another one corresponding to phonon-assisted transition from 1 to 2 succeeded by tunneling from 2 to 3. Note that for our system the probability of direct phonon-assisted transition from 1 to 3 is small compared to that of the process described by Eq. (3). In fact, the integrals involving the functions $u_1(z)$ and $u_3(z)$ are proportional to the overlap factor $\exp\{-\alpha(2w + L)\}$, where α is the inverse wave function decay length in the barrier region, w is the barrier width and L is the well width. On the other hand, the product of the overlap factors appearing in Eq. (3), $\exp(-2\alpha w)$, is much greater so that for $\exp(-\alpha L) \ll 1$ the probability of the direct transition from 1 to 3 is negligible. This is in contrast to the conventional situation in the problem of hopping between localized states, where usually the probability of direct phonon-assisted transition between distant sites is much larger than that for transitions involving virtual states.

The shunt resistance R_{13} for the processes involving virtual states is defined by (1), where the transition probabilities are expessed in terms of the modulus of the matrix element (3) squared. Provided that $(E_3 - E_1) > kT$, we obtain that the conductivity is activated with the activation energy $E_3 - \mu$, which is smaller than the activation energy $E_2 - \mu$ for the conduction mode with nearest-well transitions in the critical region. Another difference between the nearest-well and distant-well conduction modes is a different scaling behavior as the disorder energy (the width of the level distribution) is varied. As follows >from the corresponding dependence of $|c_{12}|^2$, the preexponential factor of the conductance is inversely proportional to the disorder energy squared whereas the activation energy scales as the disorder energy. Note that the usual experimentally studied structures with the number of the wells of the order 100 [4] are mesoscopic and can have widely fluctuating conductances for various realizations of the size quantized level distribution. The scaling behavior of the conductance can, however, be checked for these structures if one uses series of structures similar to those described in [4] with different disorder energies for the same random realization.

3 Discussion

The scaling behavior of the preexponential factor of the conductance is expected to become evident in the case of small activation energies. The activation energy can be small if the energies E_1, E_3 for the states localized on different sides of the critical well lie below the Fermi level. Actually, the levels E_1, E_3 can correspond to hybridized states of several wells (clusters) rather than to individual wells adjacent to the critical one. If the miniband width for the corresponding regular SL (or the transfer energy) is not too small compared with the disorder energy, then the average number of sites in the clusters is sufficiently large; then for the most probable realization of disorder the lowest energies of hybridized states of the clusters adjoining the critical well lie below the Fermi level. In this case the nonactivated temperature dependence of the vertical conductance is expected.

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References

[1] J. D. Dow, S. Y. Ren and K. Hess, *Phys. Rev. B* **25**, 6218 (1982).
[2] A. Chomette, B. Deveaud, A. Regreny and G. Bastard, Phys. Rev. Lett. **57**, 1464 (1986).
[3] T. Yamamoto, M. Kasu, S. Noda and A. Sasaki, *J. Appl. Phys.* **68**, 5318 (1990).
[4] G. Richter, W. Stolz, S. Koch, P. Thomas, K. Maschke and I. P. Zvyagin, *Superlattices and Microstructures* **22**, 475 (1997).
[5] M. Lee, S. A. Solin and D. R. Hines, *Phys. Rev. B* **48**, 11921 (1993).
[6] I. P. Zvyagin and M. A. Ormont, *The 24th Int. Conf. on the Physics of Semiconductors, August 2-7, 1998, Jerusalem, Israel* , Abstracts, p. Tu-P87.
[7] B. I. Shklovskii and A. L. Efros, *Electronic Theory of Doped Semiconductors* (in Russian), Moscow, Nauka, 1979.
[8] V. L. Bonch-Bruevich, I. P. Zvyagin, R. Keiper, A. G. Mironov, B. Esser and R. Enderlein, *Electronic Theory of Disordered Semiconductors* (in Russian), Moscow, Nauka, 1981.